

# Formule di Trigonometria

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha}$$

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$$

$$\cos \alpha = \pm \frac{1}{\sqrt{1 + \tan^2 \alpha}}$$

$$\sin \alpha = \pm \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}$$

$\alpha$	$\alpha$	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$
$0^\circ$	0	0	1	0	$\infty$
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$45^\circ$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$60^\circ$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$90^\circ$	$\frac{\pi}{2}$	1	0	$\infty$	0

Formule di addizione

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Formule di duplicazione

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 2\cos^2 \alpha - 1 \\ 1 - 2\sin^2 \alpha \end{cases}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

Formule parametriche

$$\begin{aligned} \sin \alpha &= \frac{2t}{1+t^2} \\ \cos \alpha &= \frac{1-t^2}{1+t^2} \\ \tan \alpha &= \frac{2t}{1-t^2} \end{aligned} \quad \left( t = \tan \frac{\alpha}{2} \right)$$

Formule di prostaferesi

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\sin p - \sin q = 2 \sin \frac{p-q}{2} \cos \frac{p+q}{2}$$

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}$$

Formule di bisezione

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

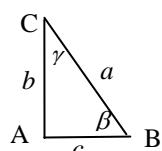
$$\tan \frac{\alpha}{2} = \begin{cases} \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \\ \frac{1 - \cos \alpha}{\sin \alpha} \\ \frac{\sin \alpha}{1 + \cos \alpha} \end{cases}$$

Formule di triplicazione

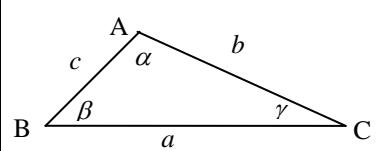
$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

Triangoli rettangoli



Triangoli qualunque



$$b = a \sin \beta = a \cos \gamma = c \tan \beta$$

$$a = \frac{b}{\sin \beta} = \frac{b}{\cos \gamma}; \tan \beta = \frac{b}{c}$$

$$c = a \sin \gamma = a \cos \beta = b \tan \gamma$$

$$a = \frac{c}{\sin \gamma} = \frac{c}{\cos \beta}; \tan \gamma = \frac{c}{b}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$\begin{aligned} \text{Area(ABC)} &= \frac{1}{2} bc \sin \alpha \\ &= \frac{1}{2} ac \sin \beta \\ &= \frac{1}{2} ab \sin \gamma \end{aligned}$$